Can growth take place while reducing emissions? The role of energy mix*

Brigitte Castañeda† **Hernando Zuleta**‡

Abstract

Do the macroeconomic effects of a carbon tax differ between countries according to the primary energy source? We answer this question using a theoretical model of directed technical change and empirically test the main results. We find four main results: (i) in the absence of subsidies, carbon taxes have a negative effect on economic growth; (ii) this negative effect is a decreasing function of the proportion of clean energy sources; (iii) subsidies for clean inputs have a positive effect on economic growth; and (iv) the magnitude of the positive effect grows with the proportion of clean energy sources. The empirical results are consistent with the predictions of the theoretical model, indicating that environmental policies should consider the initial state of the energy mix and the impact of the transition to clean sources on economic growth.

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†Corresponding author. Universidad de los Andes, Bogotá, Colombia. E-mail: k.castaneda@uniandes.edu.co

‡Universidad de los Andes, Bogotá, Colombia. E-mail: h.zuleta@uniandes.edu.co

1 Introduction

Why do some countries adopt carbon taxes with minimal economic disruption, while in others such policies face strong political and economic resistance? Macroeconomic analyses often report an average effect of carbon pricing, but this hides important differences across countries. This paper argues that a country's initial energy mix[1] —its reliance on "clean"[2] versus "dirty"[3] energy sources—is an important determinant of the macroeconomic consequences of carbon taxation.

We study this question using both theory and data. First, we build a two-sector model of directed technical change based on Acemoglu *et al.* (2012a), taking into account the initial composition of the two sectors. The model shows that a carbon tax operates as a negative productivity shock to the dominant dirty sector, making the effect on growth dependent on the initial energy mix. In fossil-fuel-dependent economies, the tax is more likely to contract output, while in economies with a stronger clean sector, it can redirect innovation and support the transition. The model also incorporates subsidies, which are often part of real-world policy packages (Marten & van Dender, 2019), and shows how they can reduce short-run costs. We then test these predictions with data from 66 countries between 1990 and 2020, using modern staggered difference-in-differences estimators.

Our empirical findings align with the model. We show that the initial energy mix strongly influences the short-run growth effects of carbon taxation. In countries with a low initial share of clean energy (below the sample median), GDP growth falls by 1.3 percentage points in the second year after the tax and by 2.8 percentage points in the third. In countries with a high initial share of clean energy, we find no significant decline in growth; instead, the point estimates are positive, up to 1.4 percentage points. These results indicate that the growth effects of carbon taxes differ sharply depending on initial conditions.

[1] the composition of energy sources used for production and consumption
[2] (e.g., hydro, nuclear, solar, and wind power)
[3] (e.g., coal, oil, and natural gas)

While foundational integrated assessment models, such as those by Nordhaus (1993), established the link between economic activity and climate, they often treated technological progress as exogenous. Subsequent work, including Stern (2007) and Romer & Romer (2010), highlighted the urgency of the climate problem but also underscored the need for models where technology can respond to policy incentives. Our paper fits within this line of literature by analyzing how the interaction between policy and the initial composition of energy sectors shapes macroeconomic outcomes.

This paper makes two contributions to the literature. First, we provide new evidence on the heterogeneous macroeconomic effects of carbon pricing. Previous studies such as Bernard *et al.* (2018) and Metcalf & Stock (2020) report small or insignificant average effects, but we show that averages conceal systematic heterogeneity. Our findings complement work such as Känzig & Konradt (2023), which documents heterogeneous price impacts in Europe, by offering a theoretical mechanism and evidence on aggregate growth from a broader global sample that includes developing countries.

Second, we link our empirical evidence to the theoretical literature on directed technical change (DTC) and environmental policy. This literature, developed by Acemoglu *et al.* (2012a), Acemoglu *et al.* (2012b), and Fried (2018), highlights the role of policy in redirecting innovation and the importance of path dependence. Empirical work in this area has mostly focused on micro-level outcomes, such as patenting responses to energy prices (Aghion *et al.*, 2012; Popp, 2002). Our paper is among the first to test the aggregate predictions of DTC models with cross-country data. By showing that the effects of a carbon tax depend on a country's position in the transition, we provide empirical support for core mechanisms in this class of models.

The paper is organized as follows. Section 2 introduces the theoretical model and its main propositions. Section 3 describes the data and empirical strategy. Section 4 reports the results, and Section 6 concludes.

2 Theoretical model

We develop a two-sector growth model with directed technical change to analyze how the macroeconomic effects of climate policy depend on a country's initial energy mix. Building on the framework of Acemoglu *et al.* (2012a) (hereafter AABH), the model features an economy where a final good is produced from substitutable "clean" and "dirty" energy inputs. Innovation is endogenous, and labor is mobile between sectors. Our central hypothesis is that the impact of climate policy is path-dependent: in a fossil-fuel-dependent economy, a carbon tax acts as a negative productivity shock, while in a cleaner economy, the same policy can accelerate the transition to a sustainable growth path.

To explore this heterogeneity, we model a policy package that combines a tax on the dirty sector (τ) with a subsidy for the clean sector, which we assume is a linear function of the tax rate, $q = \tau \cdot \phi$. This dual-instrument approach is motivated by the macroeconomic trade-offs of the green transition. A carbon tax (τ) is the primary tool for internalizing the negative externality of emissions. Yet, it can create short-run economic costs, such as output contraction and inflationary pressures [4] (NGFS, 2024). The complementary subsidy is designed to mitigate these costs and accelerate the transition by addressing a second market failure: positive knowledge spillovers in the nascent clean technology sector, which can lead to underinvestment in R&D (Acemoglu *et al.*, 2012a). Taken together, this two-instrument approach provides a framework in which carbon pricing and targeted subsidies work jointly to reduce short-term adjustment costs while reinforcing the long-run benefits of directed technological change.

This policy design reflects real-world practice. An OECD study found that approximately 65% of revenues from carbon taxes are earmarked for purposes such as green spending or are recycled back into the economy through other tax policies (Marten & van Dender, 2019).

[4] By raising energy prices, it acts as a negative supply shock, depressing output and adding upward pressure on inflation—an effect often described as "greenflation."

This motivates our focus on how subsidies can be used in conjunction with a tax.

For simplicity in our theoretical analysis, we do not impose a government budget constraint, assuming that any deficit or surplus is managed through lump-sum transfers. This allows us to isolate the main channels of interest without affecting the model's qualitative results. We now turn to the formal description of the economic environment.

2.1 Model Framework

2.1.1 Households and Environment

Each country is inhabited by a continuum of households, consisting of workers and scientists, who can freely switch sectors without incurring adjustment costs. The households have the following preferences:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t) \tag{1}$$

where C_t represents the consumption of the final good at time t, S_t denotes the quality of the environment, and $\rho > 0$ is the discount rate.

The environmental quality, $S_t \in [0, \hat{S}]$, where $\hat{S}$ is the baseline level of environmental quality without pollution [5]. Environmental quality degrades due to the production of dirty inputs at a rate $\xi > 0$ but regenerates at a natural rate $\delta > 0$. Finally, in the event of an environmental disaster, it collapses to $S_t = 0$. Therefore, evolution of environmental quality can be expressed by the following law of motion:

$$S_{t+1} = min\left[\hat{S}, (1+\delta)S_t - \xi Y_{dt}\right] \tag{2}$$

where Y_{dt} is the aggregate production of the dirty intermediate good at time t.

[5] The quality of the environment absent any human pollution.

2.1.2 Final Good

There is a unique final good, Y_t, produced competitively using "clean" and "dirty" inputs (depending on the primary energy source required) Y_c and Y_d.

$$Y_t = \left(Y_{ct}^{\frac{\epsilon-1}{\epsilon}} + Y_{dt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $\epsilon \in (0, +\infty)$ is the elasticity of substitution between the two sectors. If the inputs are (gross) substitutes, $\epsilon > 1$, then any final good production can be obtained from alternative clean energies. For example, renewable energy, provided it can be stored and transported efficiently, may replace energy derived from fossil fuels (Popp, 2002). On the contrary, if the two inputs are (gross) complements, $\epsilon < 1$, then it is impossible to produce without fossil fuels.

Final good producers choose the quantity of each input to maximize profits. Given the production function in equation 3 and the policy-adjusted prices, their problem is:

$$\max_{Y_{dt}, Y_{ct}} \{Y_t - (1+\tau)P_{dt}Y_{dt} - (1-\phi\tau)P_{ct}Y_{ct}\}$$

where Y_{dt} and Y_{ct} represent the quantities of dirty and clean inputs, respectively, and P_{dt} and P_{ct} denote their market prices. The parameter τ captures the carbon tax rate imposed on dirty inputs, while $\phi\tau$ represents a subsidy rate for clean inputs.

Under perfect competition, the price of the final good equals its marginal cost. Since the final good is the numeraire, its price is normalized to one. The dual of the CES production function gives the unit cost index, which leads to the following zero-profit condition:

$$\left[((1+\tau)P_{dt})^{1-\epsilon} + ((1-\tau\phi)P_{ct})^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1 \tag{4}$$

The tax is charged on the price paid for each unit of the dirty inputs demanded, while the subsidy is applied as a discount for each unit of clean inputs purchased. Consequently, the relative demand for dirty inputs declines due to the tax, whereas the demand for clean inputs increases as the subsidy reduces their effective prices.

2.1.3 Clean and Dirty Intermediate Inputs

The two inputs, Y_c and Y_d are produced competitively.[6] Using L_{jt} labor, A_{jit} the quality of machine i in the sector j, and x_{jit} a continuum of sector-specific machines (intermediates). Thus, the optimization problem faced by producers in both sectors involves maximizing profits through the optimal allocation of labor and machines.

$$\max_{x_{jit},L_{jt}} \left\{ P_{jt} L_{jt}^{1-\alpha} \left(\int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} \cdot di \right) - w_{jt} L_{jt} - \int_0^1 p_{jit} x_{jit} \cdot di \right\}$$

From the first-order conditions we obtain the demand for machines and labor in each sector:

$$x_{jit} = \left(\frac{\alpha P_{jt}}{p_{jit}} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \; and \; L_{jt} = \left(\frac{(1-\alpha) P_{jt}}{w_{jt}} \right)^{\frac{1}{\alpha}} A_{jit}^{\frac{1-\alpha}{\alpha}} x_{jit}. \tag{5}$$

In line with AABH we assume that machines are produced at marginal cost equal to α^2 under monopolistic competition and sold at price p_{jit}, taking into account the demand for machines x_{jit}. Therefore the profits of the monopolists, π_{jit}, are given by: $\pi_{jit} = (p_{jit} - \alpha^2) x_{jit}$. So, replacing the demand for machines, the profits of the monopolist are:

$$\pi_{jit} = (p_{jit} - \alpha^2) \left(\frac{\alpha P_{jt}}{p_{jit}} \right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \tag{6}$$

[6] In this version, we do not consider the depletion of fossil resources. Although fossil fuel reserves are finite, historical prices have not followed the predictions of the Hotelling model, and scarcity constraints are less relevant in the context of climate change targets (Fried, 2018).

As a result, each monopolist sets a price $p_{jt} = \frac{1}{\alpha}$, identical for all i. Thus, replacing the price of machine p_{jt} in equation 5, the optimal demand for machines in each sector is obtained:

$$x_{jit} = \alpha^{\frac{2}{1-\alpha}} A_{jit} L_{jt} \left(P_{jt}\right)^{\frac{1}{1-\alpha}} \tag{7}$$

Combining the previous results, we obtain the quantities of inputs produced in each sector as follows:

$$Y_{jt} = \alpha^{\frac{2\alpha}{1-\alpha}} A_{jt} L_{jt} \left(P_{jt}\right)^{\frac{\alpha}{1-\alpha}} \tag{8}$$

where $A_{jt} = \int_0^1 A_{jit} di$.

2.1.4 Endogenous and Directed Technical Change

At the beginning of every period all intermediate goods within a sector starts with the average level of productivity of the previous period. Successful scientists invent a better version of the machine i in sector j and increases the quality of the machine by a factor γ. The probability of success is $\eta_j \in (0,1)$ and depends positively on the investment on research and development (R&D), R_{jt} and inversely on the desired productivity A_{jit}: $\eta_{jt} = \lambda \left(\frac{R_{jt}}{A_{jit}}\right)^{\sigma}$. When an innovation is unsuccessful $(1-\eta_j)$, the sector's productivity is equal to that of the previous period, A_{jt-1}.

$$A_{jt} = \begin{cases} \gamma A_{jt-1} & \text{if successful } (\eta_j) \\ A_{jt-1} & \text{if not successful} (1-\eta_j) \end{cases}$$

The problem for entrepreneurs is to maximize the expected profits of innovating: $\max_{R_{jt}} \{\eta_{jt} P_{Ajit} - R_{jt}\}$. From the first-order condition and given that the price of the patent equals the net profits of the machine producer $P_{Ajit} = \pi_{jit}$, we derive the probability of innovating in each sector:

$$\eta_{jt} = 2(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} L_{jt} \tag{9}$$

2.2 Equilibrium and Directed Technical Change

We now characterize the decentralized equilibrium of the economy. A competitive equilibrium is defined by sequences of prices and allocations such that all agents optimize given prices, and all markets clear.

Definition 2.1 (Competitive Equilibrium)**.** *Given a policy* $\{\tau, \phi\}$ *and initial technologies* $\{A_{c0}, A_{d0}\}$*, a competitive equilibrium is a set of sequences for prices* $\{P_{ct}, P_{dt}\}_{t=0}^{\infty}$*, wages* $\{w_t\}_{t=0}^{\infty}$*, and allocations of inputs* $\{Y_{ct}, Y_{dt}\}_{t=0}^{\infty}$*, labor* $\{L_{ct}, L_{dt}\}_{t=0}^{\infty}$*, and machines* $\{x_{jit}\}_{t=0}^{\infty}$ *such that for all* t*:*

1. *The final good producer maximizes profits.*
2. *Intermediate good producers in each sector (*$j \in \{c, d\}$*) maximize profits.*
3. *Machine producers in each sector (*$j \in \{c, d\}$*) maximize profits.*
4. *Households supply labor inelastically, and the labor market clears:* $L_{ct} + L_{dt} = 1$*.*
5. *The final good is the numeraire, and its market clears, implying the price index condition in equation 4 holds.*

The central equilibrium condition is found by equating the relative supply and demand for intermediate inputs. Relative supply is determined by the production technology (equation 8), while relative demand stems from the final good producer's optimization. This market clearing condition links policy, technology, prices, and factor allocations:

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{P_{ct}}{P_{dt}}\right)^{\frac{\alpha}{1-\alpha}} \frac{A_{ct}}{A_{dt}} \frac{L_{ct}}{L_{dt}} = \left(\frac{P_{dt} \cdot (1+\tau)}{P_{ct} \cdot (1-\tau\phi)}\right)^{\epsilon} \tag{10}$$

To understand how policy influences the direction of innovation, we decompose this equilibrium into its three constituent forces: the *price effect*, the *market size effect*, and the *direct productivity effect*.

1. The Price Effect incentivizes innovation in the sector with a higher producer price, which translates into greater potential profits. In equilibrium, relative producer prices are determined solely by relative productivities. As shown in Appendix 7.1.2, while the policy affects the price levels, it does so proportionally, leaving their ratio unchanged:

$$\frac{P_{ct}}{P_{dt}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{(1-\alpha)} \tag{11}$$

This result shows that the technologically scarcer input is more expensive. However, since the policy does not directly alter this ratio, its primary influence must operate through the market size.

2. The Market Size Effect incentivizes innovation for the larger market, which in our model is determined by sectoral employment. To find the equilibrium labor allocation, we substitute the relative producer price (equation 11) into the market clearing condition that equates relative supply and demand (see equation 10):

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} \left(\frac{A_{ct}}{A_{dt}}\right)^{(1-\alpha)(\epsilon-1)} \tag{12}$$

Equation 12 shows how environmental policy drives the market size effect. The tax and subsidy directly reallocate labor towards the clean sector, expanding its market for innovation. This wage differential drives a reallocation of labor towards clean input production.

3. Direct Productivity Effect With equilibrium prices and labor allocations determined, the relative production of clean versus dirty inputs is given by:

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} \left(\frac{A_{ct}}{A_{dt}}\right)^{\epsilon(1-\alpha)} \tag{13}$$

We can now determine the net direction of technical change. The relative profitability of innovation depends on the combination of the price effect (equation 11), the market size effect (equation 12), and the direct productivity effect (the "standing on shoulders" effect, where innovation is more effective in the already advanced sector). Combining these forces, the relative probability of successful innovation becomes:

$$\frac{\eta_{ct}}{\eta_{dt}} = \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} \tag{14}$$

where $\varphi = (\epsilon - 1)(1 - \alpha)$. In the absence of policy, innovation is directed toward the technologically leading sector. However, the policy term $(\frac{1+\tau}{1-\phi\tau})^{\epsilon}$ can counteract this tendency, creating a "market size" large enough to pull innovation towards the clean sector, even if it is technologically behind.

Proposition 2.1 (Condition for Redirecting Technical Change)**.** *For environmental policy to successfully redirect innovation towards the clean sector (i.e., achieve $\eta_{ct} > \eta_{dt}$), the tax rate τ must be sufficiently high to overcome the technological advantage of the dirty sector. The minimum required tax rate, τ_{min}, is given by:*

$$\tau > \tau_{min} \equiv \frac{\left(\frac{A_{dt}}{A_{ct}}\right)^{\frac{\varphi-1}{\epsilon}} - 1}{1 + \phi\left(\frac{A_{dt}}{A_{ct}}\right)^{\frac{\varphi-1}{\epsilon}}} \tag{15}$$

Proof: The result follows directly by setting $\eta_{ct}/\eta_{dt} > 1$ in equation 14 and solving for τ.

■

Proposition 2.1 yields two key insights. First, the required tax is an increasing function of the technological gap (A_{dt}/A_{ct}), implying that policy intervention is more difficult in economies with a less developed clean sector[7]. Second, the required tax is a decreasing

[7]Notice that equations 3 and 13 imply that the final good output can be written in the following form:

$$Y_t = Y_{ct}\left(1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right)^{\frac{\epsilon}{\epsilon-1}} \text{ and } Y_t = Y_{dt}\left(1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\right)^{\frac{\epsilon}{\epsilon-1}}$$

function of the subsidy parameter ϕ, highlighting that subsidies make the transition less reliant on high carbon taxes.

2.3 Policy Effects on Output and Growth

Having established the mechanism of directed technical change, we now present the model's main predictions regarding the effect of environmental policy on aggregate output and economic growth. The detailed mathematical derivations are provided in Appendix 7.3 and 7.4.

2.3.1 Effect on Aggregate Output

Once the aggregate output Y_{jt} is established (see Appendix 7.2). We can express the aggregate output Y_t by substituting the sectoral production function A9 into the final good production function (equation 3), as follows.

$$Y_t = \alpha^{\frac{2\alpha}{1-\alpha}} \cdot \left((1+\tau)^{\epsilon-1} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon-1} A_{dt}^{\varphi}\right)^{\frac{\epsilon}{\epsilon-1}} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \tag{16}$$

Taking the logarithm of equation (see A12), we can notice that the introduction of subsidies reduces the adverse effect of the carbon tax.

Proposition 2.2 (Effect on Aggregate Output)**.** *The introduction of an environmental policy that redirects technical change has the following effects on the level of aggregate output (Y_t):*

1. *In the absence of subsidies ($\phi = 0$), a carbon tax has a negative effect on aggregate output ($\frac{\partial \log(Y_t)}{\partial \tau} < 0$).*
2. *This negative impact is mitigated by the presence of a subsidy for clean inputs ($\frac{\partial^2 \log(Y_t)}{\partial \tau \partial \phi} > 0$).*

Therefore, as $\frac{A_{dt}}{A_{ct}}$ decreases $\frac{Y_{ct}}{Y_t}$ grows and $lim_{\frac{A_{dt}}{A_{ct}} \to 0} \frac{Y_{ct}}{Y_t} = 1$.

3. *If the elasticity of substitution is sufficiently high ($\epsilon > \alpha/(1-\alpha)$), then along the energy transition ($Ac < Ad$), the negative effect of the tax is a decreasing function of the relative productivity of the clean sector (A_{ct}/A_{dt}).*

Proof: See Appendix 7.3 ■

Proposition 2.2 formalizes the central trade-off of climate policy. It suggests that the implementation of environmental policies can initially generate negative economic impacts, as a carbon tax acts as a negative shock to the economy by making a main production input more expensive. However, the size of this effect depends on the relative productivity of the clean sector. Economies with a higher clean-to-dirty productivity ratio (A_{ct}/A_{dt}) face smaller losses because they can substitute away from dirty inputs more easily. Appendix 7.3 provides a graphical illustration of these comparative statics, plotting the marginal effect of the tax as a function of the initial productivity ratio (see Figure 10).

The proposition also shows the importance of policy design. A tax combined with subsidies can lead to better income outcomes. Subsidies increase activity in the clean sector, while taxes reduce innovation in the dirty sector. As the clean sector expands relative to the dirty one, the overall negative effect on aggregate output declines over time. Using tax revenues to finance clean energy subsidies helps manage the short-run costs of the transition.

2.3.2 Effect on Economic Growth

While the short-term effects of climate policy on output levels can be negative, the long-term effects on the *growth rate* can be positive. The aggregate growth rate of the economy is a weighted average of the growth rates of the clean and dirty sectors:

$$\frac{\Delta Y_t}{Y_t} = \left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \cdot \frac{\Delta Y_{ct}}{Y_{ct}} + \left(\frac{Y_{dt}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}} \cdot \frac{\Delta Y_{dt}}{Y_{dt}} \tag{17}$$

Environmental policy impacts this aggregate growth rate through two distinct channels, both of which favor the clean sector. First, the policy increases the relative size of the clean sector (Y_{ct}/Y_t), giving it a larger weight in the aggregate growth calculation. Second, by redirecting innovation, the policy increases the growth rate of the clean sector ($\Delta Y_{ct}/Y_{ct}$) relative to the dirty sector. The following proposition summarizes the conditions under which these effects lead to higher overall economic growth.

Proposition 2.3 (Effect on Economic Growth)**.** *If the clean sector becomes sufficiently large relative to the dirty sector ($Y_{ct} > Y_{dt}$), and if the elasticity of substitution between inputs is high enough such that innovation is strongly path-dependent ($\varphi > 1$), then environmental policy has a positive effect on the long-run aggregate growth rate of the economy* $\frac{\partial(\Delta Y_t/Y_t)}{\partial\tau} > 0$. *We establish this proposition by combining two intermediate effects:*

- *From equation 13, it implies $Y_{ct}/Y_{dt} > 1$. The clean sector is the larger sector.*
- *If $\varphi > 1$, from equation 14, it also implies $\eta_{ct} > \eta_{dt}$. Innovation is directed to the clean sector.*

Proof: See Appendix 7.4. ■

The intuition behind Proposition 2.3 is that once the policy has successfully redirected innovation and shifted the economic structure, the economy's primary growth engine becomes the clean sector. Since innovation is now concentrated in what has become the dominant part of the economy, this targeted technological progress translates into a higher aggregate growth rate. This highlights the potential for a "green growth" path, where a well-designed climate policy package not only addresses environmental externalities but can also become a driver of long-term economic expansion by accelerating innovation in the industries of the future. A graphical representation of the conditions for positive growth effects is provided in Appendix 7.4 (see Figure 11).

2.4 Calibrated Simulations

To quantify the model's key mechanisms, we conduct calibrated simulations. This approach shows the model's predicted dynamics using empirical initial conditions. The model is calibrated to represent two stylized economies based on countries that adopted carbon taxes at similar times but had vastly different energy structures: a high-carbon economy, based on Japan's pre-tax energy mix in 2011 (13.5% clean energy share) before its 2012 tax; and a low-carbon economy, based on France's mix in 2013 (48.4% clean energy share) prior to its 2014 tax.

Table 1: Model Calibration

Parameter		**Value**	**Source / Justification**
Panel A: Externally Set Parameters			
Substitution Elasticity	ϵ	3.0	Acemoglu *et al.* (2012a)
Capital Share	α	0.33	Standard macro value
Innovation Efficiency	γ	1.0	Normalization
Baseline Growth Rate	g_{BGP}	1.5%	Target growth
Panel B: Calibrated Parameters			
		High-Carbon (Japan)	**Low-Carbon (France)**
Targets			
Clean Energy Share	s_c	13.5%	48.4%
Results			
Initial Prod. Ratio	A_{c0}/A_{d0}	0.254	0.953
BGP Innovation Rate	η_d	0.015	0.015

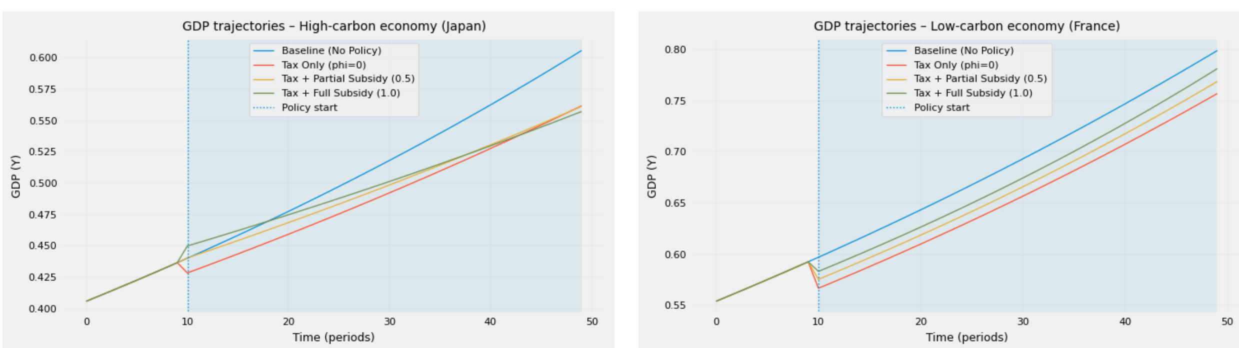
Notes: Panel A reports externally set parameters. Panel B shows calibrated parameters chosen to match data targets (clean energy share and baseline growth rate).

We follow a standard calibration strategy, setting common parameters—such as the elasticity of substitution (ϵ) and capital share (α)—to values found in the literature. For each economy, we calibrate the key country-specific parameter, the initial relative productivity of the clean sector (A_{c0}/A_{d0}), so that it exactly matches the observed clean energy share at baseline. This approach ensures empirically grounded starting points. Before policy intervention, we assume that both economies share a common long-run growth rate in their balanced growth paths. Table 1 presents all parameters and their calibrated values.

Simulations run for 50 periods (years) for each economy. At period t=10, a permanent 20 percent carbon tax (=0.2) is introduced, which raises fossil fuel costs. The analysis considers four scenarios: a no-policy baseline; the tax alone (=0); and the tax paired with either a partial (=0.5) or full (=1.0) subsidy, where the subsidy supports clean energy production.

Figure 1 presents simulated GDP trajectories that support Proposition 2.2. While the red line in the high-carbon economy (Panel A) shows a significant, persistent GDP decline from the tax alone—reflecting substantial output costs in economies reliant on fossil fuels—partial (orange line) and full (green line) subsidies substantially reduce output losses. By contrast, the low-carbon economy (Panel B) demonstrates greater resilience: with a higher initial clean technology level, policy costs decrease. The tax's initial output decline is smaller, and a full subsidy nearly eliminates the short-run negative impact.

Figure 1: Simulated Effect of Climate Policy on GDP Levels



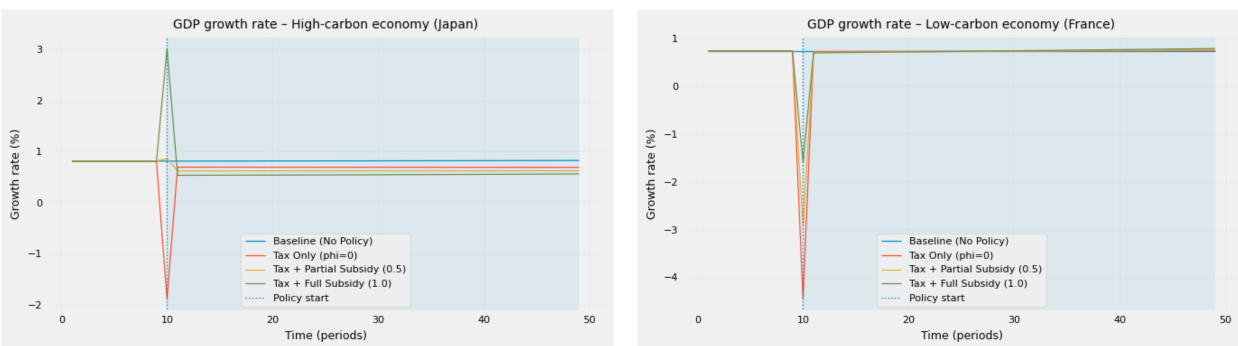
High-Carbon Economy (e.g., Japan) Low-Carbon Economy (e.g., France)

Notes: The figure shows simulated paths for Log GDP. The policy is introduced at period 10.

Figure 2 shows the growth dynamics underlying these level effects. Policy implementation at t=10 creates an immediate recessionary shock. Growth rates sharply decline. This effect is stronger in the high-carbon economy, which faces a deeper contraction. At period t=11, there is a temporary growth spike. This does not represent a traditional recovery. Instead, it reflects directed technical change. The policy prompts a rapid shift in research and development from fossil fuels to the clean sector. This reallocation creates a one-time growth surge as the new technological paradigm is adopted.

After the initial volatility, both economies settle into new long-run growth paths. Path dependency is clear: the high-carbon economy stabilizes below its original baseline, illustrating the long-term cost of shifting innovation away from fossil fuels. In contrast, the low-carbon economy's growth rate returns to baseline after a brief transition, showing that economies already focused on clean technologies can adopt climate policy without sacrificing long-run growth. This outcome supports Proposition 2.3.

Figure 2: Simulated Effect of Climate Policy on GDP Growth Rates



High-Carbon Economy (e.g., Japan) Low-Carbon Economy (e.g., France)

Notes: The figure shows simulated paths for the GDP growth rate (in percent). The policy is introduced at period 10.

In summary, the simulations confirm the model's main mechanisms. Carbon tax costs are significant and path-dependent. Fossil fuel-reliant economies face the greatest burden. Effective policies, such as redirecting tax revenues to clean energy subsidies, can reduce these costs. This approach helps economies transition to sustainable long-run growth.

3 Data and Empirical Strategy

3.1 Data

This section details the data and empirical strategy used to test the hypotheses derived from our theoretical framework. Specifically, we aim to analyze the relationship between a carbon tax and income and GDP growth. We use data from the World Bank Group, Energy Consumption data from the International Energy Agency, and Carbon pricing data from the World Carbon Pricing Database.

Table 2: Description of the main outcome variables.

Variable	**Mean**	**Median**	**Std. Dev.**	**Source**
Real GDP (millions US$ constant 2017)	1026887	258975	2511953	Penn World Table
Crecimiento del PIB (anual %)	2.86%	2.99%	4.33%	Data WorldBank
GDP per capita (current US$)	9.384	9.532	1.143	Data WorldBank
Employment rate (% total labor)	92.30%	92.94%	4.56%	Data WorldBank
Population, total	49035868	9771437	165987702	Data WorldBank
Primary energy consumption (TWh)	1589	324	4390	Our World in Data
Clean energy fraction* (% total consumption)	14%	9%	16%	International Energy Agency
Clean electricity fraction* (% total consumption)	37%	32%	31%	International Energy Agency
Countries	66			
Observations	2044			

**Primary sources of clean energy are hydro, nuclear, solar, and wind power.*

We use a yearly data panel from a sample of 66 countries, of which 23 had implemented a carbon tax. The sample covers the period from 1990 to 2020. Table 2 presents descriptive statistics of the variables of interest and the sources of the databases. The outcome variables studied are GDP growth (%) and the employment rate measured as the number of employees over an economically active population (%). Table 4 in the annex presents in detail the characteristics of the carbon tax for each of the countries studied. It can be seen that since

2010 the adoption of carbon taxes in the countries has increased. The table also shows the carbon tax's monetary value as well as the percentage of emissions covered by the tax. To apply the theoretical model to real-world data, we use the share of primary energy consumed by each source as a proxy for the initial rates of production of clean and dirty inputs, Y_c and Y_d. We categorized the sample of countries based on the share of primary energy consumed by each source, dividing them into those with a low-carbon intensity energy mix and those with a high-carbon intensity energy mix. The term "low-carbon intensity" is used to describe the energy consumption of hydro, nuclear, solar, and wind sources. These sources emit lower levels of carbon than traditional fossil fuels. Conversely, the term "high-carbon intensity" is used to describe energy generated from the combustion of fossil fuels, such as coal, oil, natural gas, and biofuels. Figure 3 shows the share of energy from low-carbon intensity sources by countries.

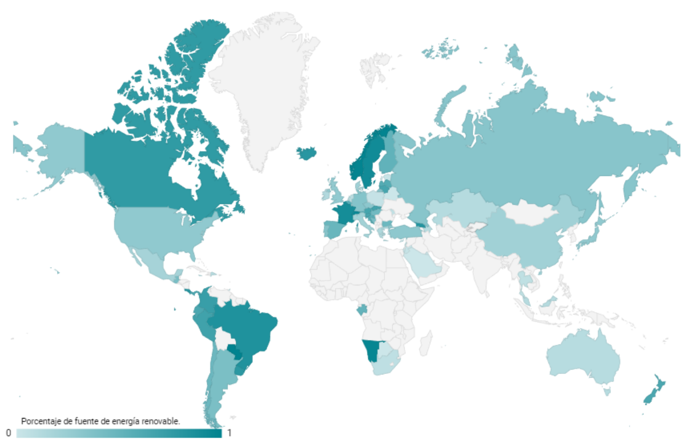


Figure 3: Map of countries according to the share of clean energy sources

The countries with a share of clean energy higher than average (14%), at the time of implementation of the carbon tax, constitute the database of countries with a "low-carbon intensity" energy matrix. Similarly, the countries that had a clean energy share lower than average (14%) at the time of implementing the carbon tax constitute the database of countries with a "high-carbon intensity" energy matrix. Table 3 presents the statistics of the outcome variables for each sample, the full sample of 66 countries, and the sample of countries with polluting and clean energy mix.

3.2 Empirical strategy

In this paper, we estimate the effect of introducing a carbon tax on GDP growth, according to the primary energy sources of consumption, to validate the corollaries defined in the theoretical model. For this, we use the event study method with the estimators proposed by Callaway & Sant'Anna (2021), Sun & Abraham (2021), and TWFE (Two Way Fixed Effects).

Event Study is used to estimate the effect of introducing a carbon tax on GDP growth rate. In this approach, we aim to estimate the effect on GDP which is not associated with its historical economic growth. We assume that changes in GDP not predicted by historical GDP growth in the country itself, nor by current and past international economic shocks, are exogenous. Several studies have used the event study strategy to analyze the effects of regulatory changes on carbon prices, energy, and stock prices (Mansanet-Bataller & Pardo (2009); Fan *et al.* (2017); Bushnell *et al.* (2013), among others).

We introduce the following assumptions, which capture the effect of carbon tax on GDP: *Treatment timing.* The treatment time assumption refers to a scenario in which there are several periods and countries implement a carbon tax at any time within those periods. Once a country implements the tax, it remains in treatment for the remainder of the period. This assumption implies that the timing of the implementation of the tax is unrelated to other factors such as GDP that may influence the outcome, meaning that it is considered exogenous. In other words, the timing of tax implementation is independent of other factors and is not influenced by them.

No-anticipation assumption. The implementation of the carbon tax does not affect the path of GDP outcomes before the treatment period. In other words, the counterfactual outcome paths for GDP in periods before the treatment period would have been the same whether or not the carbon tax had been implemented at some point in the future. Similarly, the treatment assignment does not depend on the potential GDP outcomes in any period.

Parallel trends. The parallel trends assumption in the context of a staggered events study with a carbon tax as the treatment and GDP as the outcome variable would imply that in the absence of the carbon tax, the trends in GDP would be parallel across the treated and control groups. In this study, any differences in the post-treatment outcomes between the two groups can be attributed to the treatment (i.e., the carbon tax) and not to pre-existing differences in the trends of GDP.

3.2.1 Heterogeneous Effects

Proposition 2 states that the effect of a carbon tax on the economy's growth rate is negative if the polluting sector's share in final output exceeds a critical level relative to the share of the clean sector. On the other hand, it indicates that the carbon tax promotes economic growth if the share of the clean sector in final production surpasses a critical level relative to the polluting sector. To test this hypothesis empirically, we use the Event Study strategy and examine two sub samples of countries. The first sub-sample consists of "polluting countries," where the share of clean sources is below the country average. The second sub-sample includes "clean countries", where the share of clean sources exceeds the country average (above 14%).

In this model, we consider the year in which the carbon tax was introduced as year 0. We then define the periods before ($t < 0$) and after ($t > 0$) the introduction of the carbon tax, and we align time t=0 for all countries in the treatment group. We assume that the evolution of the potential outcome in the absence of the treatment can be decomposed into a time-fixed effect. Based on this assumption, we estimate the average dynamic effect of introducing a carbon tax on GDP growth ($Y_{c,t}$) in country c and year t. To conduct our analysis, we employ equation 18.

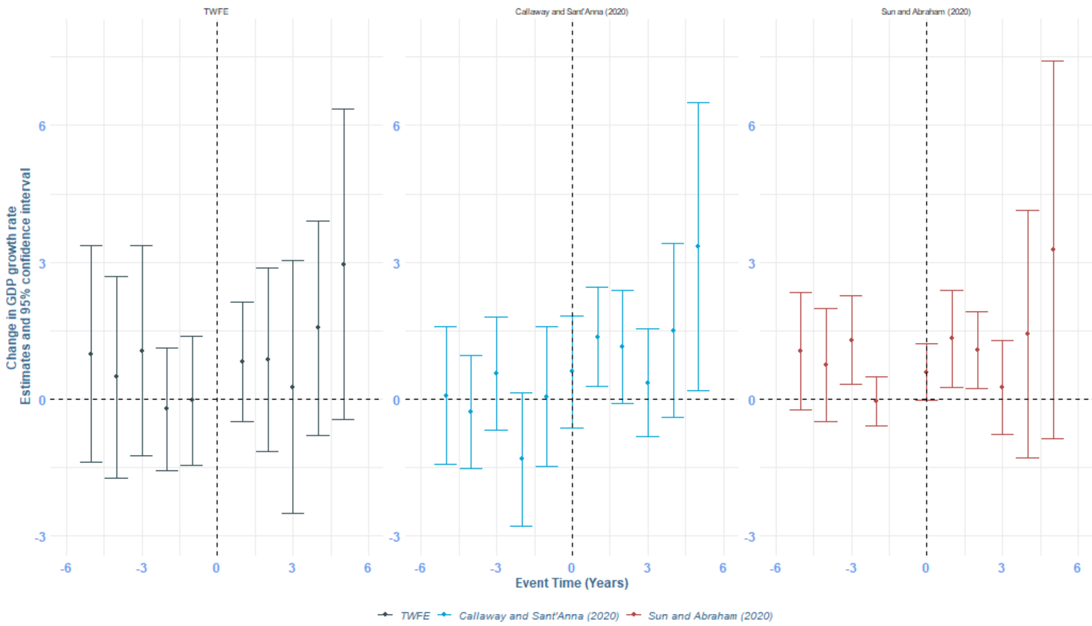
$$Y_{c,t} = \beta_1 \sum_{-T \leq r \leq T}^{r \neq 0} 1 \left[CarbonTax_{c,t} = r \right] + \Phi_c + \Phi_t + \epsilon_{c,t} \tag{18}$$

where $Y_{c,t}$ is the GDP growth rate. β_1 measures the average dynamic effect of a carbon tax in the sample of countries. When the outcome variable is the GDP growth rate this estimator tests the *proposition 2*. Also include α_c country fixed effects c for unobserved country-specific characteristics and Φ_t time fixed effects to capture other policy and time-varying resource price shocks, among other changes that may occur over time.

4 Results

We begin by examining the average dynamic effect of a carbon tax on GDP growth across our full sample of 66 countries. As shown in Figure 4, the point estimates suggest a small, positive, but often statistically insignificant effect in the years following implementation. This average result, however, can be misleading as it masks significant heterogeneity, a core prediction of our theoretical framework. The model predicts that the impact of a carbon tax is not uniform but is instead critically dependent on a country's initial energy mix. Therefore, we now turn to test our main hypotheses by analyzing the effects across different subgroups of countries.

Figure 4: Average Dynamic Effects of Carbon Tax on GDP Growth (Full Sample)



Notes: The figure displays the dynamic treatment effects of carbon tax implementation on the annual GDP growth rate using the full sample of 66 countries. Estimates are shown from different staggered adoption estimators: Callaway and Sant'Anna (CS), Sun and Abraham (SA), and Two-Way Fixed Effects (TWFE). The vertical bars represent 95% confidence intervals.

4.1 Heterogeneous Effects: Testing the Model's Predictions

In this section, we test the core predictions of our theoretical model regarding the heterogeneous effects of carbon taxation. We split our sample based on the share of clean energy in the primary energy mix at a pre-treatment baseline. This allows us to directly test our hypotheses about how initial conditions mediate the policy's impact.

4.1.1 Hypothesis 1: Negative Impact in High-Carbon Economies

Consistent with our first hypothesis (H1), which posits that a carbon tax will have an initial negative effect on growth in economies heavily reliant on fossil fuels, we analyze the subsample of "high-carbon" countries. These are countries where the initial share of clean energy is below the sample median.

Figure 5: Annual GDP growth (%) in countries with a **high-carbon** energy mix

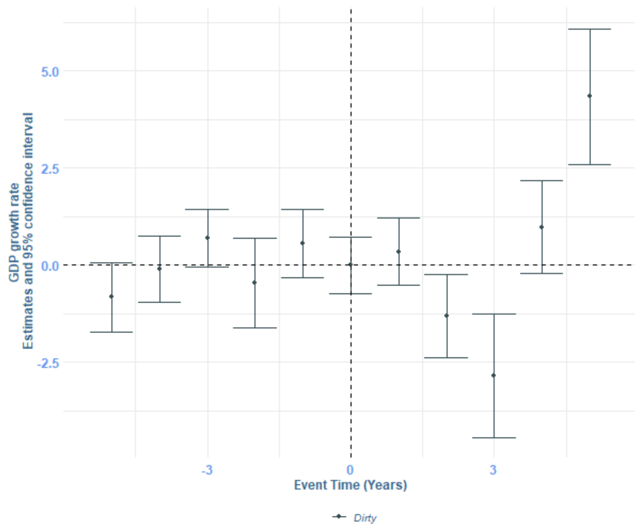


Figure 5 presents the event study estimates for this group. The results strongly support our hypothesis. It is observed that implementing the carbon tax in countries with a polluting energy matrix is associated with a statistically significant reduction in the growth rate in the second and third years following the policy. In the second year, the cumulative effect of implementing the carbon tax is -1.3 percentage points of annual GDP; in the third year, the effect is -2.8 percentage points. This finding aligns with the mechanism proposed in our model: by increasing the cost of the dominant dirty inputs, the tax acts as a short-term negative productivity shock, thereby discouraging innovation and investment in the economy's primary sector. Interestingly, the effect appears to fade in the long run, which could suggest a slow adaptation as the economy begins to transition towards cleaner technologies.

4.1.2 Hypothesis 2: Mitigated or Positive Impact in Low-Carbon Economies

We now turn to our second key hypothesis (H2), which predicts that this negative impact is mitigated, or even becomes positive, in countries with a larger initial share of clean energy. To test this, we examine the subsample of "low-carbon" countries—those with an initial clean energy share above the median.

Figure 6: GDP growth (%) in countries with a **low-carbon** energy mix

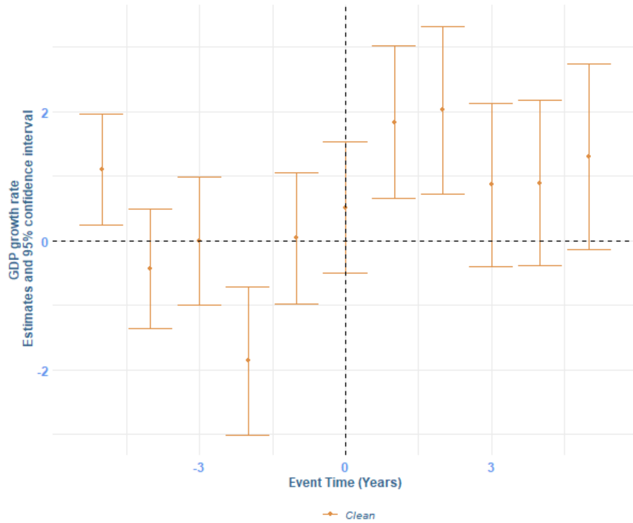


Figure 6 and the corresponding estimates in Table 5 (column 2) show that the effect of a carbon tax on GDP growth is indeed positive for this group of countries. This result suggests that introducing a carbon tax is associated with an increase in annual GDP of 1.4 percentage points in the first five years after implementing the policy, and this effect is significant. The contrast with the high-carbon group is stark and provides compelling evidence for the moderating role of the initial energy mix. The results confirm that the higher the share of clean sources at the time of implementing the climate policy, the more favorable the effect of the tax on the GDP growth rate.

4.1.3 Exploring the Threshold: A Continuous View of Heterogeneity

The binary split between "high-carbon" and "low-carbon" countries confirms our main hypotheses. However, to provide a more nuanced view, Figure 7 visualizes how the dynamic effects of the tax evolve as we continuously increase the threshold for what defines a "clean" energy mix.

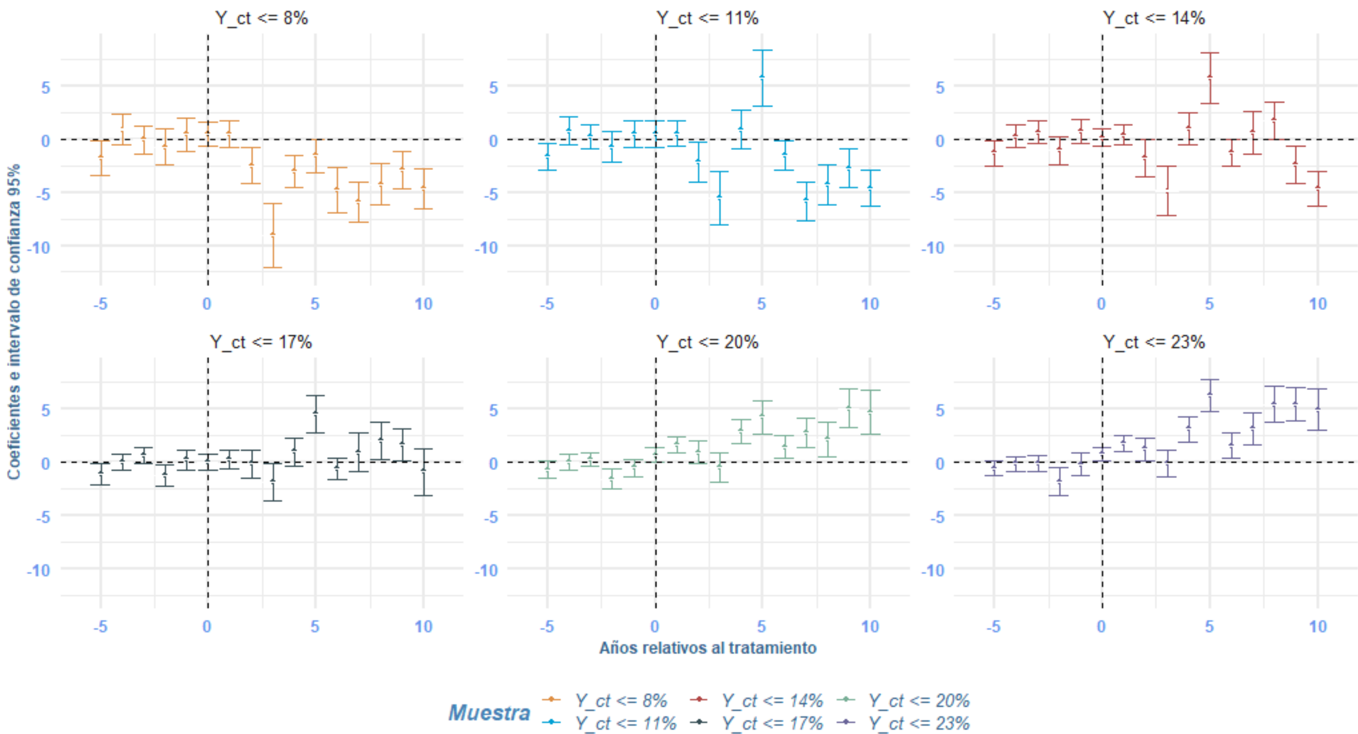


Figure 7: Dynamic effects of carbon tax across increasing clean energy thresholds

Note: This figure displays the estimated dynamic effect of carbon tax implementation on GDP growth across different subsamples. Each subplot represents a subsample defined by an increasing threshold on the proportion of clean primary energy sources in the country's energy mix. The threshold increases from countries with $\leq 8\%$ clean energy (top-left) to those with $\leq 23\%$ clean energy (bottom-right).

Observing the figure, a clear trend emerges: as the proportion of clean energy in a country's energy mix increases (moving from the top-left to the bottom-right panels), the estimated effect of the carbon tax in the years immediately following implementation (particularly years 1 and 2) shifts systematically from negative to positive. This visualization acts as a powerful robustness check, demonstrating that our findings are not an artifact of a specific sample split. It confirms that the composition of a country's energy sources is a key continuous moderator of the short-term economic impacts of carbon taxation.

5 Robustness exercises

To verify whether the previous results were robust, we performed several econometric exercises. we used the sources of electricity generation, in exchange for the energy matrix, as they can approximate the share of the sectors (clean and polluting) in the final production.

We assess the impact of the carbon tax on growth rates by examining different samples of polluting and clean countries. The goal is to identify the threshold at which the carbon tax's effect shifts from negative (for polluting countries) to positive. We defined various cut-off points Θ based on the proportion of clean energy sources in the total energy consumption. Specifically, we considered scenarios where the sample of polluting countries does not exceed certain percentages of clean energy sources. The findings on the carbon tax's impact on GDP growth, relative to these cut-off points for clean and polluting samples, are presented in 7. When the sample of polluting countries includes less than 8% clean energy (representing the most polluting countries), the effect is highly negative. Conversely, when the sample reaches 23% clean energy, the effect becomes positive. The carbon tax is beneficial for GDP growth in countries with an energy matrix comprising at least 17% clean energy.

5.1 Effect using electricity mix

The electricity mix is composed of the set of sources available to generate the electricity consumed within a country. Electricity unlike energy can be generated entirely by renewable sources, therefore, for this exercise, we divide the sample of polluting and clean countries based on the 37% share of clean sources. That is, if the country generates more than 37% of electricity from sources such as solar, wind, hydro, and nuclear, it is considered clean, and would be part of the sub-sample of clean countries, otherwise it would belong to the sub-sample of polluting countries.

Figure 8: Effect of carbon tax on GDP growth rate in countries with polluting electricity mix.

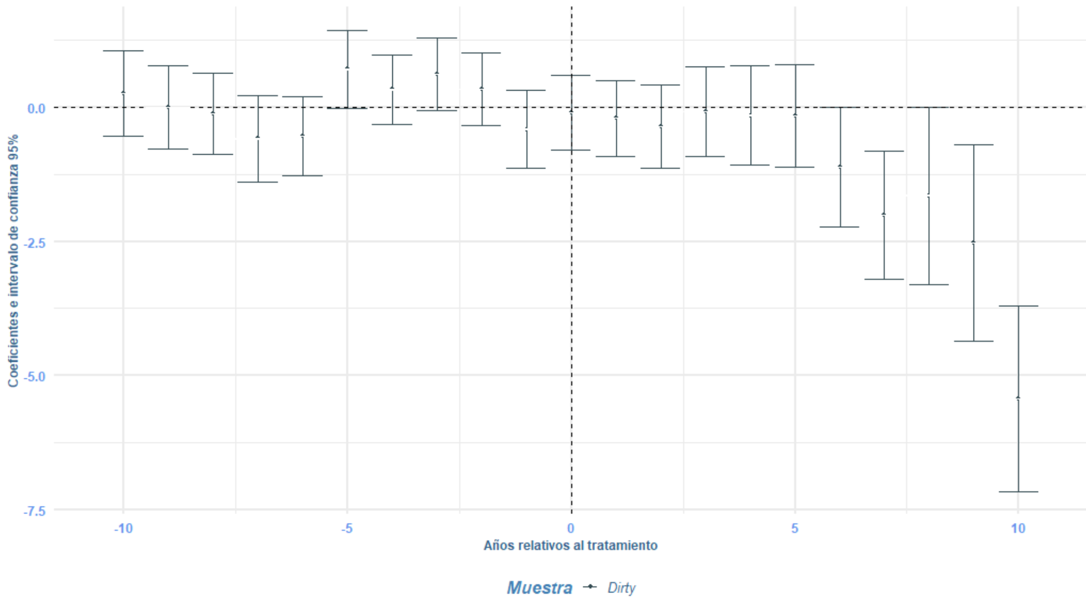
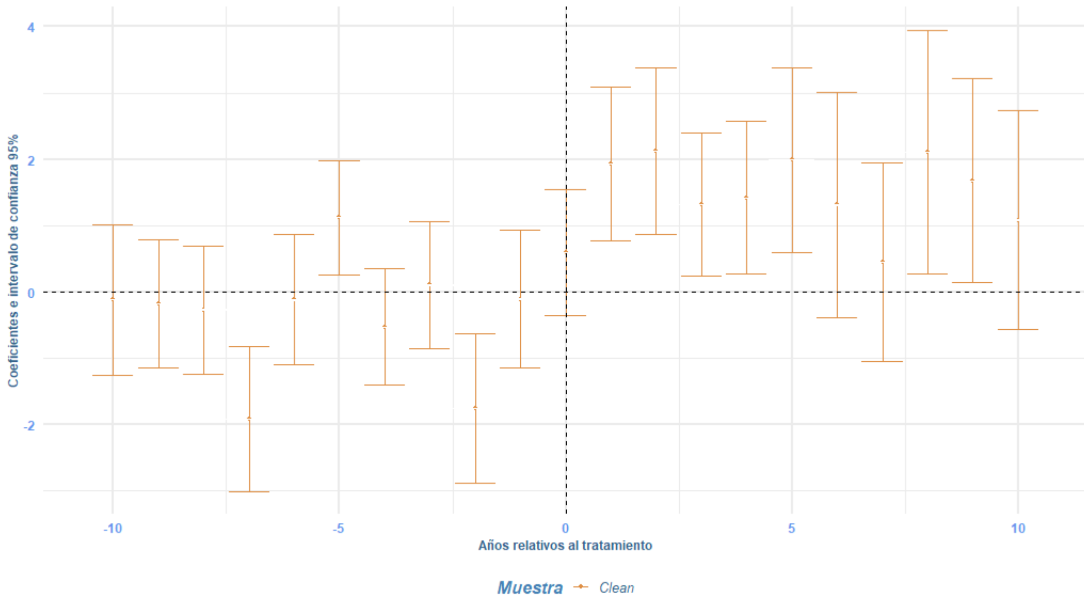


Table 7 presents the estimators of the effect of the carbon tax on the GDP growth rate according to the electricity matrix. The coefficients estimated with the electricity matrix are similar in magnitude and direction to those estimated with the energy matrix, however, these results are significant in more periods unlike those estimated with the energy matrix. Figure 8 presents the coefficients of the effect of the carbon tax on the growth rate using the sample of countries with a polluting electricity matrix. The carbon tax is associated with negative growth rates, this effect is larger in magnitude than the one calculated with the sample of countries divided according to the energy matrix. On the other hand, the figure 9 shows similar results to those obtained with the sample of clean countries using the energy matrix. For countries with a clean electricity matrix, the effect is slightly positive, increasing the growth rate by 0.6 percentage points in the first 5 years after implementing the climate policy.

Figure 9: Effect of carbon tax on GDP growth rate in countries with clean electricity mix.



6 Conclusions

This study provides robust evidence that the macroeconomic impact of a carbon tax is significantly influenced by the composition of a country's energy mix. By examining the proportion of energy generated from fossil fuels and low-carbon-intensity sources, we highlight the heterogeneity in outcomes following the implementation of a carbon tax. In economies heavily reliant on fossil fuels, the introduction of a carbon tax may lead to a short-term decline in GDP growth, as predicted by the theoretical model. However, the long-term trajectory suggests that growth can recover, particularly as the share of clean energy increases or energy efficiency improves, validating proposition 1 of the model.

Conversely, in countries where energy production relies primarily on low-carbon sources, the imposition of a carbon tax may positively impact GDP growth in the short term, with minimal or no negative effects on employment. This suggests that countries with cleaner energy mixes are better positioned to absorb the initial economic costs of carbon pricing and can even experience economic benefits from the transition to cleaner production.

Our findings also indicate that the adverse effects on GDP growth in high-carbon-intensity economies tend to dissipate over time. This is due to a shift in demand away from polluting goods, which incentivizes innovation and expansion in the clean energy sector. As this sector grows, it eventually overtakes the polluting industries, allowing the economy to return to its pre-tax growth trajectory. The transition is marked by a reallocation of labor and capital towards cleaner technologies, driven by the carbon tax's effect of increasing the relative cost of polluting goods.

The study also supports the idea that a carbon tax can serve as an effective policy tool not only for reducing carbon emissions but also for fostering long-term clean economic growth. As the tax increases the costs of production in high-emission sectors, it simultaneously encourages greater productivity and innovation in the clean sector, ultimately transforming the economic structure towards sustainability.

A key policy implication derived from our model is the strategic use of carbon tax revenues. In the early stages of the transition, it is important to reinvest these revenues in the development and scaling of clean technologies, enabling the clean energy sector to meet growing demand. This reinvestment can mitigate the short-term negative impact on economic growth, facilitating a smoother transition to a low-carbon economy.

In conclusion, the study underscores the importance of considering a country's energy mix when designing carbon taxes and other climate policies. Tailoring these policies to national contexts can optimize their economic and environmental effectiveness, minimizing transitional costs while accelerating the shift towards sustainable development.

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Supplementary Tables

Table 5: Effect of carbon tax on GDP growth rate, according to energy matrix

Effect on GDP growth rate				
Period	Estimator	Standard error	Estimator	Standard error
	Panel A. Clean Countries		*Panel B. Dirty Countries*	
-4	-0.4407	(0.4502)	-0.1065	(0.4174)
-3	-0.006	(0.5013)	0.7044	(0.3898)
-2	-1.8644*	(0.5847)	-0.4588	(0.5817)
-1	0.0342	(0.4994)	0.5648	(0.4513)
0	0.5112	(0.5215)	-0.0016	(0.3975)
1	1.8328*	(0.6228)	0.3466	(0.4201)
2	2.024*	(0.6683)	-1.3099*	(0.5608)
3	0.8623	(0.6068)	-2.8622*	(0.764)
4	0.8891	(0.6215)	0.9814	(0.5686)
5	1.3018	(0.733)	4.3461*	(0.9005)
6	0.7619	(0.872)	-0.4006	(0.5029)

Note: Panel A shows the results for the sample of cleaner countries, which consists of 25 countries, while Panel B presents the results for the more polluting countries, which consists of 41 countries.

Table 3: Descriptive statistics of the samples

Variable	**Mean**	**Median**	**Std. Dev.**	**Mean**	**Median**	**Std. Dev.**
All countries						
	With carbon tax			without carbon tax		
GDP real (millions 2017US$)	850686.90	393482.88	1082446.42	1121275.39	166135.61	3006622.00
GDP per capita (current US$)	9.89%	10.10%	0.94%	9.11%	9.15%	1.15%
GDP growth (annual %)	2.482	2.683	3.694	3.058	3.254	4.630
Employment rate (% total labor)	92.07%	92.85%	4.75%	92.43%	93.02%	4.45%
Primary energy consumption (TWh)	1094.63	474.96	1379.31	1855.63	254.93	5334.58
Clean electricity fraction (%)	47%	46%	33%	31%	24%	29%
Clean energy fraction (%)	23%	18%	20%	9%	4%	11%
Countries	23			43		
Countries with a low carbon intensity energy mix						
	With carbon tax			without carbon tax		
GDP real (millions 2017US$)	609238.88	322000.91	721705.56	654034.94	161623.54	1096710.94
GDP per capita (current US$)	10.02%	10.17%	0.91%	9.26%	9.44%	1.04%
GDP growth (annual %)	2.17	2.57	3.28	2.35	2.67	3.47
Employment rate (% total labor)	92.14%	92.70%	4.51%	90.99%	91.68%	4.23%
Primary energy consumption (TWh)	841.80	351.14	1103.98	748.81	232.82	1166.34
Clean electricity fraction (%)	70%	71%	21%	60%	60%	16%
Clean energy fraction (%)	36%	32%	17%	23%	23%	8%
Countries	13			12		
Countries with a high carbon intensity energy mix						
	With carbon tax			without carbon tax		
GDP real (millions 2017US$)	1164569.32	579572.91	1359162.31	1302519.86	168376.80	3459270.47
GDP per capita (current US$)	9.72%	9.86%	0.95%	9.06%	9.07%	1.19%
GDP growth (annual %)	2.89	2.83	4.14	3.33	3.57	4.98
Employment rate (% total labor)	91.97%	93.96%	5.06%	93.06%	93.67%	4.39%
Primary energy consumption (TWh)	1423.29	1010.65	1614.45	2289.49	277.86	6198.64
Clean electricity fraction (%)	16%	12%	15%	19%	10%	24%
Clean energy fraction (%)	6%	5%	6%	3%	1%	5%
Countries	10			31		

Note: "Clean electricity fraction" includes electricity from renewables and nuclear. "Clean energy fraction" encompasses clean electricity and direct renewable energy use. Units are as indicated in the variable names, with GDP in millions of 2017 US$ and primary energy consumption in TWh.

Table 4: Characteristics of the carbon tax in the countries analyzed.

Jurisdiction	**Sectors**	**Fuels**	**Tax Point**	**Year**	**Emissions**	**Covered**	**Price**
Argentina	Most	Liquid, gas	Prod., distr.	2018	441	20%	6
Canada	Most	Fossil	Reg. distr.	2019	817	22%	32
Chile	Power, ind.	Fossil	Users	2017	149	39%	5
Colombia	Most	Liquid, gas	Sellers, importers	2017	190	24%	5
Denmark	Build., transp.	Fossil	Distr.	1992	63	35%	28
Estonia	Power, ind.	Thermal	Users	2000	28	6%	2
Finland	Ind., transp.	Fossil excl. peat	Distr.	1990	112	36%	72.8
France	Ind., build., transp.	Fossil	Distr.	2014	488	35%	52
Iceland	Most (ETS)	Liquid, gas	Prod., distr.	2010	5	55%	35
Ireland	Most (ETS)	Fossil	Distr.	2010	65.6	49%	39
Japan	Most	Fossil	Prod., distr.	2012	1345	75%	3
Latvia	Ind., power	Fossil excl. peat	Distr.	2004	18	3%	14
Liechtenstein	Ind., build., transp.	Fossil	Distr.	2008	0	26%	101
Mexico	Power, ind., etc.	All excl. nat. gas	Prod., distr.	2014	822	23%	3
Norway	Most (ETS)	Liquid, gas	Prod., distr.	1991	75	66%	69
Poland	Most (ETS)	Fossil	Users	1990	429	4%	0.08
Portugal	Ind., build.	Fossil	Distr.	2015	81	29%	28
Singapore	Power, ind.	Fossil	Facility ops.	2019	56	80%	4
Slovenia	Build., transp.	Fossil	Distr.	1996	21	50%	20
South Africa	Ind., build., etc.	Not spec.	Users	2019	640	80%	9
Spain	F-gases	Not spec.	First entry	2014	367	3%	18
Sweden	Transp., build. (ETS)	Fossil	Distr.	1991	111	40%	137
Switzerland	Ind., build., etc.	Fossil	Distr.	2008	55	33%	101
United Kingdom	Power	Fossil	Users	2013	583	23%	25
Ukraine	Ind., build.	Fossil	Users	2011	312	71%	0.3

Note: Emissions in MtCO_2e; coverage as share of national GHG emissions; prices in US$.
Source: World Bank's State and Trends of Carbon Pricing report.

Table 6: Effect of carbon tax on GDP growth rate.

Effect on GDP growth rate			
Period	TWFE	Sun et. al.	Callaway et. al.
-4	0.548 (1.1334)	0.7931 (0.625)	-0.151 (0.6237)
-3	1.1046 (1.1724)	1.297** (0.4847)	0.5186 (0.6281)
-2	-0.1466 (0.6834)	0.0445 (0.2604)	-1.2647 (0.7162)
-1	-0.0484 (0.7365)	0.6092. (0.2996)	-0.0448 (0.7143)
0	0.9087 (0.6737)	1.543** (0.572)	0.6128 (0.6395)
1	1.0405 (1.047)	1.326*** (0.4413)	1.5846** (0.5919)
2	0.4668 (1.4567)	0.5076 (0.5355)	1.3926* (0.5856)
3	1.7464 (1.2217)	1.707 (1.363)	0.605 (0.6133)
4	3.1444 (1.7416)	3.575 (2.085)	1.7872. (0.9386)
5	0.7838 (1.0323)	1.027 (0.6695)	3.651* (1.6256)
6	0.7748 (1.4977)	1.021 (1.232)	1.1508 (0.6942)
Fixed-Effects			
Country	Yes	Yes	No
Year	Yes	Yes	No
S.E.:Clustered	Country	Country	Country
Observations	2288	2288	2288

Note: This table presents estimators of the average effect of implementing a carbon tax on GDP growth rate over a ten-year event window. Columns TWFE, Sun et al., and Callaway et al. show the estimated effect using different econometric specifications. The values represent the percentage point change in GDP growth rate associated with the implementation of a carbon tax.

Table 7: Effect of carbon tax on GDP growth rate, according to electricity matrix composition.

Effect on GDP growth rate				
Period	Estimator	Standard error	Estimator	Standard error
	Panel A. Clean Countries		*Panel B. Dirty Countries*	
-4	-0.5326	(0.4392)	-0.1527	(0.4358)
-3	0.1009	(0.4992)	0.7496	(0.4114)
-2	-1.7654*	(0.5608)	-0.4363	(0.5449)
-1	-0.111	(0.5012)	0.4642	(0.4953)
0	0.5908	(0.4877)	-0.0125	(0.4074)
1	1.9277*	(0.599)	0.2502	(0.4246)
2	2.1262*	(0.6625)	-1.4785*	(0.569)
3	1.317*	(0.566)	-2.956*	(0.8038)
4	1.4174*	(0.6158)	0.8687	(0.6318)
5	1.9908**	(0.7196)	4.2354*	(0.9454)
6	1.3091	(0.8273)	-0.5926	(0.5206)

Note: This table shows the estimated effect of carbon taxes on GDP growth rate, differentiated by the electricity matrix composition of countries. Panel A presents results for countries with a clean electricity matrix, while Panel B shows results for countries with a dirty electricity matrix. The estimators are similar to those obtained using the energy matrix, with some differences in significance. For countries with a dirty electricity matrix, the effect tends to be negative, whereas for countries with a clean electricity matrix, the effect is slightly positive, increasing the growth rate by approximately 0.6 percentage points in the first five years after carbon tax implementation.

7 Appendix: Mathematical Derivations

7.1 Derivation of Equilibrium Allocations and Prices

This section provides the detailed derivation of the main equilibrium objects used in the main text.

7.1.1 Intermediate Inputs

From the first-order conditions is obtained the demand for machines and labor in each sector,

$$x_{jit} = \left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \quad \text{and} \quad L_{jt} = \left(\frac{(1-\alpha)P_{jt}}{w_{jt}}\right)^{\frac{1}{\alpha}} A_{jit}^{\frac{1-\alpha}{\alpha}} x_{jit} \tag{A1}$$

where w_{jt} denotes the wage paid for each unit of labor hired and p_{jit} is the price that the producer of inputs must pay for each machine used.

Producers of intermediate goods maximize profits by knowing the demand function they face,

$$\max_{x_{ji}} \{p_{jit} x_{jit} - x_{jit}\} \tag{A2}$$

Machines are produced at marginal cost ν under monopolistic competition and, sold at price p_{jt}, taking into account the demand for machines x_{jit} in the sector in which they are used. Therefore the profits of the monopolists, π_{jt}, are given by: $\pi_{jt} = (p_{jit} - \nu)x_{jit}$. So, replacing the demand for machines, the profits of the monopolist are:

$$\pi_{jt} = (p_{jit} - \nu)\left(\frac{\alpha P_{jt}}{p_{jit}}\right)^{\frac{1}{1-\alpha}} A_{jit} L_{jt} \tag{A3}$$

Following Acemoglu *et al.* (2012a), we normalize $\nu = \alpha^2$, so, each monopolist sets a price $p_{jit} = \frac{1}{\alpha}$. Thus, replacing the price of machine $p_{jit} = \frac{1}{\alpha}$ in equation A1, the optimal demand for machines and the profits of intermediate goods in each sector can be written as:

$$x_{jit} = \alpha^{\frac{2}{1-\alpha}} A_{jit} L_{jt} \left(P_{jt}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \pi_{jt} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} A_{jit-1} L_{jt} \tag{A4}$$

and the quantities of inputs produced in sector j are:

$$Y_{jt} = \alpha^{\frac{2\alpha}{1-\alpha}} A_{jt} L_{jt} \left(P_{jt}\right)^{\frac{\alpha}{1-\alpha}} \tag{A5}$$

Combining this equation A3 and replacing ν, the equilibrium profits of machine producers can be written as:

$$\pi_{jt} = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} P_{jt}^{\frac{1}{1-\alpha}} A_{jit-1} L_{jt} \tag{A6}$$

7.1.2 Factors of production in equilibrium

From the profit-maximization problem of the producer of machines, assuming the total labor supply is normalized to one, such that $L_{ct} + L_{dt} = 1$, and given that equilibrium wages are equal, $w_{ct} = w_{dt}$, we can substitute the price index (equation 4) to express the equilibrium labor allocation in each sector as follows:

$$\begin{aligned} L_{ct} &= \frac{(1+\tau)^{\epsilon} A_{ct}^{\varphi}}{(1-\tau\phi)^{\epsilon} A_{ct}^{\varphi} + (1+\tau)^{\epsilon} A_{dt}^{\varphi}} \\ L_{dt} &= \frac{(1-\tau\phi)^{\epsilon} A_{dt}^{\varphi}}{(1-\tau\phi)^{\epsilon} A_{ct}^{\varphi} + (1+\tau)^{\epsilon} A_{dt}^{\varphi}} \end{aligned} \tag{A7}$$

where $\varphi = (\epsilon - 1)(1-\alpha)$. Additionally, the equilibrium prices can be determined as follows:

$$\begin{aligned} P_{ct} &= \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\tau\phi)^{-(\epsilon-1)} A_{dt}^{\varphi}\right)^{\frac{1}{\epsilon-1}}}{A_{ct}^{(1-\alpha)}} \\ P_{dt} &= \frac{\left((1+\tau)^{-(\epsilon-1)} A_{ct}^{\varphi} + (1-\tau\phi)^{-(\epsilon-1)} A_{dt}^{\varphi}\right)^{\frac{1}{\epsilon-1}}}{A_{dt}^{(1-\alpha)}} \end{aligned} \tag{A8}$$

It is important to note that subsidies for clean production increase the prices of both inputs, while taxes on dirty production decrease them. However, because the percentage change is proportional for both prices, the relative price ratio remains unchanged. Regarding labor equilibrium, higher productivity and the carbon tax in the clean sector lead to a greater allocation of labor to that sector. Furthermore, as the clean energy subsidy increases, the labor share in the clean sector expands, even if the technological level in the clean sector is relatively low.

7.2 Static Effects of Environmental Policy on Sectoral Production

To analyze the effect of the policy on aggregate output, we first express aggregate output Y_jt in each sector j in terms of fundamental parameters. Replacing the prices (A8) and labor (A7) in equation 8, we obtain the output of the two sectors in terms of productivity, as well as the tax and subsidy rates.

$$\begin{aligned} Y_{ct} &= \alpha^{\frac{2\alpha}{1-\alpha}} \cdot (1+\tau)^{\epsilon} A_{ct}^{\epsilon(1-\alpha)} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \\ Y_{dt} &= \alpha^{\frac{2\alpha}{1-\alpha}} \cdot (1-\phi\tau)^{\epsilon} A_{dt}^{\epsilon(1-\alpha)} \cdot \frac{\left(\frac{A_{ct}^{\varphi}}{(1+\tau)^{(\epsilon-1)}} + \frac{A_{dt}^{\varphi}}{(1-\phi\tau)^{(\epsilon-1)}}\right)^{\frac{\alpha}{\varphi}}}{(1+\tau)^{\epsilon} A_{ct}^{\varphi} + (1-\phi\tau)^{\epsilon} A_{dt}^{\varphi}} \end{aligned} \tag{A9}$$

Equation A9 indicates that the tax and the subsidy affect the production of both inputs. Now, in order to identify the direction and magnitude of the effect of the environmental policy, we derive the production with respect to τ.

$$\begin{aligned} \frac{\partial \log(Y_{ct})}{\partial \tau} =& \frac{\epsilon}{1+\tau} - \frac{\alpha}{1-\alpha} \left(\frac{(1+\tau)^{-\epsilon} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{-\epsilon}}{(1+\tau)^{-(\epsilon-1)} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}} \right) \\ & - \epsilon \left(\frac{(1+\tau)^{\epsilon-1} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon} \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon}} \right) \end{aligned} \tag{A10}$$

$$\frac{\partial \log(Y_{dt})}{\partial \tau} = -\frac{\epsilon\phi}{1-\phi\tau} - \frac{\alpha}{1-\alpha}\left(\frac{(1+\tau)^{-\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{-\epsilon}}{(1+\tau)^{-(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{-(\epsilon-1)}}\right) - \epsilon\left(\frac{(1+\tau)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} - \phi(1-\phi\tau)^{\epsilon-1}}{(1+\tau)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + (1-\phi\tau)^{\epsilon}}\right) \tag{A11}$$

From equations A10 and A11:

- The effect of a tax on dirty inputs is greater (more positive or less negative) for the clean sector, i.e., $\frac{\partial \log(Y_{ct})}{\partial \tau} > \frac{\partial \log(Y_{dt})}{\partial \tau}$. In other words, a tax on dirty inputs generates a sectoral redistribution in favor of the clean sector at the expense of the dirty sector.

- For low levels of relative productivity of the clean sector, $\frac{A_{ct}}{A_{dt}}$, an increase in the tax rate results in an increase in the production of clean inputs. Specifically, if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}$, then $\frac{\partial \log(Y_{ct})}{\partial \tau} > 0$.

- For high levels of relative productivity of the clean sector, an increase in the tax rate leads to a decrease in the production of dirty inputs. Specifically, if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > \phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}$, then $\frac{\partial \log(Y_{dt})}{\partial \tau} < 0$. Therefore, if fiscal policy is strong enough to generate this transition, it will also have a negative effect on the production of dirty inputs. In particular, if $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} > 1$, then $\frac{\partial \log(Y_{dt})}{\partial \tau} < 0$.

- The second derivative with respect to the relative productivity of the clean sector, $\frac{A_{ct}}{A_{dt}}$, is always negative, and exactly the same for both sectors: $\frac{\partial^2 \log(Y_{jt})}{\partial \tau \partial \frac{A_{ct}}{A_{dt}}} < 0$ for $j = \{d, c\}$. This implies that increasing the relative productivity of the clean sector negatively affects the impact of the tax on output for both sectors.

7.3 Proof of Proposition 2.2 (Effect on Aggregate Output)

Taking the logarithm of the aggregate output Y_t (equation 16), differentiating with respect to τ, and rearranging the expression for the marginal effect of the tax, we obtain:

$$\begin{aligned}\frac{\partial \log(Y_t)}{\partial \tau} = & \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{1}{(1+\tau)}\epsilon\left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}\right) \\ & - \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} \frac{1}{(1+\tau)}\frac{\alpha}{(1-\alpha)}\left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}}\right) \\ & -\phi\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}\frac{1}{(1+\tau)}\epsilon\left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}} - \frac{\left(\frac{1+\tau}{1-\phi\tau}\right)}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}}\right) \\ & +\phi\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\frac{1}{(1+\tau)}\frac{\alpha}{(1-\alpha)}\left(\frac{1}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1-\phi\tau}{1+\tau}\right)^{-(\epsilon-1)}}\right)\end{aligned} \tag{A12}$$

Notice that the sum of the terms third and fourth is positive. Therefore, the introduction of subsidies reduces the negative effect of the carbon tax.

Proof of Claim 1 (Negative effect without subsidy): Setting $\phi = 0$ in equation A12 simplifies the expression to:

$$\begin{aligned}\frac{\partial \log(Y_t)}{\partial \tau} = & \frac{\epsilon}{(1+\tau)}\left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1}{1+\tau}\right)^{\epsilon-1}} - \frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1}{1+\tau}\right)^{\epsilon}}\right) \\ & - \frac{\alpha}{(1-\alpha)}\frac{1}{(1+\tau)}\left(\frac{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1}{1+\tau}\right)^{-(\epsilon-1)}}\right)\end{aligned}$$

and

$$\frac{\partial \log(Y_t)}{\partial \tau} = -\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\frac{1}{(1+\tau)}\left(\frac{\epsilon\tau\left(\frac{1}{1+\tau}\right)^{\epsilon}}{\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1}{1+\tau}\right)^{\epsilon-1}\right)\left(\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1}{1+\tau}\right)^{\epsilon}\right)} + \frac{\frac{\alpha}{(1-\alpha)}}{\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}+\left(\frac{1}{1+\tau}\right)^{-(\epsilon-1)}}\right)$$

Therefore $\frac{\partial \log(Y_t)}{\partial \tau} < 0$.

Proof of Claim 2 (Mitigation by subsidy): It follows directly from equation A12. The objective of environmental policy, in this context, is to ensure that the productivity of the clean sector is at least as high as that of the polluting sector, so that production factors flow into the clean sector and technological progress is greater in this sector. We have just seen that carbon taxes have a negative effect on income levels. In the following lines, we demonstrate that, over the course of the transition, this negative effect diminishes as the relative productivity of the clean sector increases. Specifically, as long as the productivity of the clean sector is lower than that of the polluting sector, the negative impact of the environmental policy decreases as the relative productivity of the clean sector grows. Additionally, once the productivity of the two sectors is equal, the effect of environmental policy on income becomes positive.

Proof of Claim 3 (Role of relative productivity): To prove that the negative effect of the tax is a decreasing function of the clean sector's productivity, we analyze the second derivative $\frac{\partial^2 \log(Y_t)}{\partial \tau \partial (A_{ct}/A_{dt})}$.

$$\frac{\partial^2 \log(Y_t)}{\partial \tau \partial \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} = \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} \left((1+\tau)^{-1} - \phi(1-\phi\tau)^{-1}\right)$$
$$\left(\frac{\epsilon}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\right]^2} - \frac{\frac{\alpha}{1-\alpha}}{\left[\left(\frac{1-\phi\tau}{1+\tau}\right)^{(\epsilon-1)}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + 1\right]^2}\right)$$
$$+ \frac{1}{(1+\tau)^2}\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1} \epsilon \left(\frac{2\tau\phi}{\left[\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon}\right]^2}\right)$$

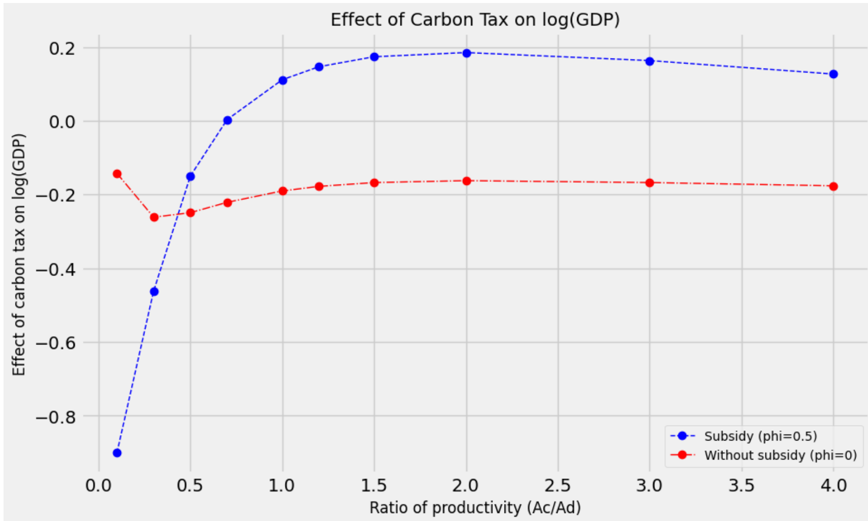
Two main observations are determined:

- If $\epsilon > \frac{\alpha}{1-\alpha}$ and $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} < 1$, then the combined effect of the tax rate change and the productivity ratio between sectors results in a positive increase in aggregate output growth, specifically $\frac{\partial^2 \log(Y_t)}{\partial \tau \partial \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}} > 0$. This implies that, under this condition, growth is enhanced when the clean sector's productivity relative to the dirty sector is low.
- If $\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} = 1$ and $\epsilon > \frac{\alpha}{1-\alpha}$, then the direct effect of the tax on output is positive, meaning $\frac{\partial \log(Y_t)}{\partial \tau} > 0$. This suggests that when both sectors have equal productivity, the tax application benefits output growth.

This shows that as the clean sector becomes more productive, the negative impact of the tax diminishes.

Figure 10 provides a graphical illustration of these results. It plots the marginal effect of the carbon tax on the logarithm of GDP, $\partial \log(Y_t)/\partial \tau$, as a function of the relative productivity of the clean sector, A_{ct}/A_{dt}. The figure is generated using equation A12 and the baseline parameters from our calibration.

Figure 10: Static Effect of Carbon Tax on log(GDP)



Notes: The figure shows the marginal effect of the tax on log(GDP) for different levels of relative clean productivity (A_{ct}/A_{dt}). The red line shows the effect without a subsidy ($\phi = 0$), and the blue line shows the effect with a partial subsidy ($\phi = 0.5$). The parameters used are $\epsilon = 3$, $\alpha = 0.3$, and $\tau = 0.9$.

The figure visually corroborates the claims of the proposition. The red line (no subsidy) is always in the negative region, confirming Claim 1. The blue line (with subsidy) is consistently above the red line, demonstrating the mitigating effect of the subsidy as stated in Claim 2. Finally, both lines are upward sloping for $A_{ct}/A_{dt} < 1$, showing that the negative effect of the tax diminishes as the clean sector's productivity catches up, and the effect can even turn positive under a policy mix for a sufficiently clean economy, as established in Claim 3.

7.4 Proof of Proposition 2.3 (Effect on Economic Growth)

The aggregate growth rate is given by equation 17. The policy's effect on this rate operates through its impact on sectoral shares and sectoral growth rates. (i) the effect on the sectoral shares of income, $\frac{\partial \frac{Y_{ct}}{Y_t}}{\partial \tau} > 0$ and $\frac{\partial \frac{Y_{dt}}{Y_t}}{\partial \tau} < 0$; (ii) the effect on the growth rate of the two sectors, $\frac{\partial \frac{\Delta Y_{ct}}{Y_t}}{\partial \tau} > 0$ and $\frac{\partial \frac{\Delta Y_{dt}}{Y_t}}{\partial \tau} < 0$.

1. Effect on Sectoral Shares: The policy shifts the composition of output in favor of the clean sector. The sectoral shares are given by:

$$\begin{aligned}\frac{Y_{ct}}{Y_t} &= \left[1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}} \\ \frac{Y_{dt}}{Y_t} &= \left[1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}}\end{aligned} \tag{A13}$$

Differentiating with respect to τ shows that $\frac{\partial(Y_{ct}/Y_t)}{\partial\tau} > 0$:

$$\begin{aligned}\frac{\partial\left(\frac{Y_{ct}}{Y_t}\right)}{\partial\tau} &= \epsilon\left[1 + \left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-1}\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}-1}\left(\frac{A_{dt}}{A_{ct}}\right)^{\varphi}\left(\frac{1-\phi\tau}{1+\tau}\right)^{\epsilon-2}\cdot\frac{1+\phi}{(1+\tau)^2}. \\ \frac{\partial\left(\frac{Y_{dt}}{Y_t}\right)}{\partial\tau} &= -\epsilon\left[1 + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\right]^{\frac{-\epsilon}{\epsilon-1}-1}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-2}\cdot\frac{1+\phi}{(1-\phi\tau)^2}\end{aligned} \tag{A14}$$

2. Effect on Sectoral Growth Rates: The sectoral growth rates depend on the rate of technological progress in each sector. Using equation (A9), the growth rate for each sector is:

$$\begin{aligned}\frac{\Delta Y_{ct}}{Y_{ct}} &= \alpha\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}A_{dt}^{\varphi}}\right) - \varphi\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}}\right) + \epsilon(1-\alpha)\frac{\Delta A_{ct}}{A_{ct}} \\ \frac{\Delta Y_{dt}}{Y_{dt}} &= \alpha\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi} + \left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon-1}A_{dt}^{\varphi}}\right) - \varphi\left(\frac{A_{ct}^{\varphi}\frac{\Delta A_{ct}}{A_{ct}}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}\frac{\Delta A_{dt}}{A_{dt}}}{A_{ct}^{\varphi}\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon} + A_{dt}^{\varphi}}\right) + \epsilon(1-\alpha)\frac{\Delta A_{dt}}{A_{dt}}\end{aligned} \tag{A15}$$

The difference in sectoral growth rates is directly proportional to the difference in innovation success rates: $\frac{\Delta Y_{ct}}{Y_{ct}} - \frac{\Delta Y_{dt}}{Y_{dt}} = \epsilon(1-\alpha)\gamma(\eta_{ct} - \eta_{dt})$. Since the policy is designed to ensure $\eta_{ct} > \eta_{dt}$, it follows that the growth rate of the clean sector will be higher than that of the dirty sector.

3. Proof of the Proposition 2.3: The proposition states that if the conditions are met, $\frac{\partial(\Delta Y_t/Y_t)}{\partial\tau} > 0$. We establish this by combining the effects. The condition $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$

from the proposition implies two intermediate results:

- From equation 13, it implies $Y_{ct}/Y_{dt} > 1$. The clean sector is the larger sector.
- If $\varphi > 1$, from equation 14, it also implies $\eta_{ct} > \eta_{dt}$. Innovation is directed to the clean sector.

Claim 1: *$\frac{Y_{ct}}{Y_{dt}} > 1$ and $\eta_{ct} > \eta_{dt}$*

- *Case 1:* If $\frac{A_{ct}}{A_{dt}} > 1$ and $\varphi > 1$ then $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ and $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} > 1$.
- *Case 2:* If $\frac{A_{ct}}{A_{dt}} < 1$ and $\varphi > 1$ then $\left(\frac{A_{ct}}{A_{dt}}\right)^{\phi-1} > \left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi}$ so if $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ then $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi-1} > 1$.

Therefore, $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_{dt}}\right)^{\varphi} > 1$ and $\varphi > 1$ imply $Y_t^c > Y_t^d$ and $\eta_{ct} > \eta_{dt}$.

Claim 2: $\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} > \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau}$

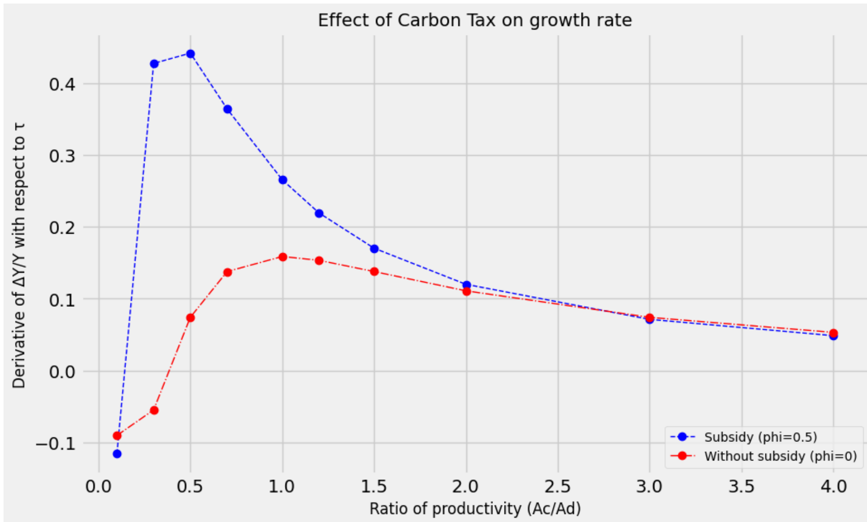
From equation A15, $\frac{\Delta Y_{ct}}{Y_{ct}} - \frac{\Delta Y_{dt}}{Y_{dt}} = \epsilon(1-\alpha)\gamma(\eta_{ct}-\eta_{dt})$ so, $\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} - \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau} = \epsilon(1-\alpha)\frac{\partial(\eta_{ct}-\eta_{dt})}{\partial\tau}$ which implies that $\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} > \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau}$.

Claim 3: $\frac{\partial\left(\frac{\Delta Y_t}{Y_t}\right)}{\partial\tau} > \left(\frac{\partial\left(\left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}}\right)}{\partial\tau}\varphi\gamma(\eta_{ct}-\eta_{dt})\right) + \left(\frac{Y_{ct}}{Y_t}\right)^{\frac{\epsilon-1}{\epsilon}}\left(\frac{\partial\left(\frac{\Delta Y_{ct}}{Y_{ct}}\right)}{\partial\tau} - \frac{\partial\left(\frac{\Delta Y_{dt}}{Y_{dt}}\right)}{\partial\tau}\right)$. From claims 1, 2 and 3 it follows that if $\left(\frac{1+\tau}{1-\phi\tau}\right)^{\epsilon}\left(\frac{A_{ct}}{A_t^d}\right)^{\varphi} > 1$ and $\varphi > 1$, then $\frac{\partial\left(\frac{\Delta Y_t}{Y_t}\right)}{\partial\tau} > 0$ and $\frac{\partial\left(\frac{\Delta Y_t}{Y_t}\right)}{\partial\tau} > 0$.

Since the policy (i) increases the weight of the faster-growing clean sector and (ii) increases the growth rate of the clean sector relative to the dirty sector, the overall effect on the aggregate growth rate is positive under the stated conditions.

Figure 11 illustrates the conditions under which the environmental policy can have a positive effect on the growth rate. The figure plots the difference between the growth rate under a given policy and the baseline growth rate, as a function of the relative productivity of the clean sector.

Figure 11: Dynamic Effect of Carbon Tax on Economic Growth



Notes: The figure shows the change in the aggregate growth rate due to the policy, relative to the baseline growth rate. A positive value indicates that the policy accelerates economic growth. The parameters used are $\epsilon = 3$, $\alpha = 0.3$, and $\tau = 0.9$.

The figure demonstrates that the effect on growth is highly dependent on the state of the economy. For low levels of clean productivity ($A_{ct}/A_{dt} < 1$), the policy may slow down growth as resources are shifted away from the dominant, high-productivity dirty sector. However, once the clean sector becomes sufficiently productive and large (for $A_{ct}/A_{dt} > 1$), the policy's effect on growth becomes positive. This is because innovation is now being directed towards what has become the main engine of the economy. The figure also shows that subsidies amplify this positive growth effect, allowing the "green growth" phase to be reached earlier and with greater magnitude. This visually confirms the conditions outlined in Proposition 2.3.